Collective and Interparticle Interactions in Even-Even Nuclei. II*

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The collective model with interparticle interactions included is used to examine the modifications of the vibrational spectra as the strength of (a) the two-particle interaction and (b) the vibration-particle interaction is varied. The $E2$, $M1$ mixing ratio δ as well as ratios of γ-ray transition probabilities are presented. These results are corrections and extensions of previous work reported.

I. INTRODUCTION

IN an earlier paper[1] the author examined the predictions of the weak-coupling collective model when interparticle interactions were included. Unfortunately, a sign error was present in the matrix for the $I=2$ states[2] so that the locations of these levels were not correctly computed. This paper presents the corrected results along with a number of γ-ray transition probability predictions. (The formalism has been completely developed in I so that only the final formulas will be given here.) In addition, the entire spectrum of levels has been computed including the levels with $I=1$, 3, 5, 7, and 8, which were omitted in the previous work. The entire calculation was done on computers to minimize error and they have been spot checked against similar calculations performed independently by MacDonald[3] at Aldermaston.

The previous calculations were limited in size to 20×20 matrices so that some of the three-phonon states could not be included. The present results include all states of one, two, and three phonons.

II. BRIEF DESCRIPTION OF PRESENT CALCULATIONS

The Hamiltonian used[4] can be broken up into two parts (a) a shell-model part, $\bar{H}$ and (b) a surface-interaction part H_s. The system treated is that of two equivalent $f_{7/2}$ particles coupled with phonons, quanta of surface excitation. [The results are equally applicable to $g_{7/2}$ particles with a modified H to give the same values of $\langle((\frac{7}{2})^2)_J|\bar{H}|((\frac{7}{2})^2)_J\rangle$.]

Harmonic oscillator radial wave functions were used to evaluate $\bar{H}$ which was taken as $\bar{H}=H_{12}=-3D\hbar\omega(3-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\exp(-r^2/r_0^2)$, with $r_0\sim2.7\times10^{-13}$ cm. The surface interaction part of the Hamiltonian, H_s, was taken as

$$H_s/\hbar\omega=\tfrac{5}{2}+\sum_\mu b_\mu^* b_\mu-(2\hbar\omega C)^{-1/2}\sum_{i=1}^{N}k(r_i)(G_i+G_i^\dagger),$$

where (1) b_μ^* and b_μ are the creation and destruction operators for phonons of spin S (where $S=2$), and $S_z=\mu$; (2) $k(r_i)$ is the radial function for the ith nucleon that determines the strength of the coupling; (3) $\hbar\omega$ is the energy of a phonon; (4) C is the surface-deformation parameter in the surface potential energy; (5) $G_i^\dagger$ is the Hermitian conjugate of G_i; and (6) $G_i\equiv\sum_\mu b_\mu Y_{2\mu}(\theta_i,\phi_i)$, where $Y_{2\mu}(\theta_i,\phi_i)$ is the normalized spherical harmonic of the angular coordinates of the ith particle. The

TABLE I. Energy of the first excited state (E_2) in units of $\hbar\omega$.

D \ x	0	0.25	0.45	0.65	1.0
0.2	0.1512	0.1984	0.2537	0.2913	0.3444
1.0	0.7560	0.6078	0.4795	0.4131	0.3952
2.0	1.0	0.8527	0.6687	0.5354	0.4497
3.0	1.0	0.9154	0.7678	0.6243	0.4963

TABLE II. Computed values, for various values of x and D, of the ratios of the reduced transition probabilities $B(E2)$ for the low-lying γ-ray transitions and the value of $E2/M1$ mixing ratio δ. In the table, δ is given in units of $E_\gamma ZA^{2/3}(\hbar\omega/2C)^{1/2}(g_j-g_R)^{-1}\times6.4\times10^{-2}$, where E_γ is the γ-ray energy in MeV, $\hbar\omega$ is the one-phonon energy, C is the potential energy parameter for surface deformation, and (g_j-g_R) is given in Table III.

D	x	$E_{2'}/E_2$	$\frac{B(E2;2'\to2)}{B(E2;2\to0)}$	$\frac{B(E2;2'\to0)}{B(E2;2\to0)}$	δ
0.2	0	6.61	...	...	...
	0.25	5.23	0.0927	2.712	0.0211
	0.45	4.50	0.7073	0.5568	0.1155
	0.65	4.35	0.7686	0.2408	0.2376
	1.00	4.50	0.6902	0.1578	0.3504
1.0	0	1.32	...	...	...
	0.25	1.92	0.1707	0.8819	0.0251
	0.45	2.67	0.9349	0.2656	0.1251
	0.65	3.24	0.9411	0.1523	0.2602
	1.00	3.95	0.7726	0.1288	0.3784
2.0	0	1.51	...	...	...
	0.25	1.89	0.6413	0.0706	0.1003
	0.45	2.22	1.506	0.0531	0.3414
	0.65	2.65	1.227	0.0695	0.4626
	1.00	3.46	0.898	0.0952	0.5228
3.0	0	2.00	2.0	...	...
	0.25	2.01	1.950	0.0012	1.941
	0.45	2.11	1.775	0.0107	1.083
	0.65	2.39	1.455	0.0310	0.9177
	1.00	3.11	1.026	0.0692	0.7828

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[1] B. James Raz, Phys. Rev. **111**, 1116 (1959); this will be referred to as I.

[2] I am indebted to Dr. N. MacDonald for pointing this error out to me.

[3] N. MacDonald (private communication).

[4] See reference 1 for a complete description of the formalism; the original investigation of this type was performed by G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).

variable x is defined in the usual manner[5] as $x=k(5/16\pi j\hbar\omega C)^{1/2}$ where $k=\langle|ki|\rangle$.

The formulas for computing the γ-ray matrix elements are standard and are given explicitly in I.

III. DISCUSSION OF THE THEORETICAL RESULTS

In order to facilitate comparison with experiments the results of the energies are normalized to the energy of the first excited state so that the curves show the ratio of the energy of a given state to that of the first excited state. Table I gives the computed energies of the first excited state in terms of $\hbar\omega$, the one-phonon energy. Table II gives the γ-ray transition probabilities for some of the observed transitions. In addition it gives the computed values of the $E2/M1$ mixing ratio δ for the $2' \rightarrow 2$ transition. The definition of δ is:

$$\delta=\frac{\sqrt{3}}{10}\left(\frac{E_\gamma}{\hbar c}\right)\frac{\langle\|M(E2)\|\rangle}{\langle\|M(M1)\|\rangle}.$$

The table lists δ in units of $E_\gamma Z A^{2/3}(\hbar\omega/2C)^{1/2}\times(g_j-g_R)^{-1}\times 6.4\times 10^{-2}$, where E_γ is given in MeV, $\hbar\omega$ is the one phonon energy, C is the constant in the

TABLE III. The gyromagnetic ratio g_j of a particle of $j=\frac{7}{2}$ with $l=3$ and 4 for a proton and a neutron. The gyromagnetic ratio g_R associated with the vibrational motion of the core and the combination of the two terms (g_j-g_R) that enter into the definition of δ (see Table II).

	g_j	g_R	(g_j-g_R)
$f_{7/2}$ neutron	−0.5465	0.40	−0.9465
$f_{7/2}$ proton	1.6550	0.40	1.2550
$g_{7/2}$ neutron	0.4251	0.40	0.0251
$g_{7/2}$ proton	0.4906	0.40	0.0906

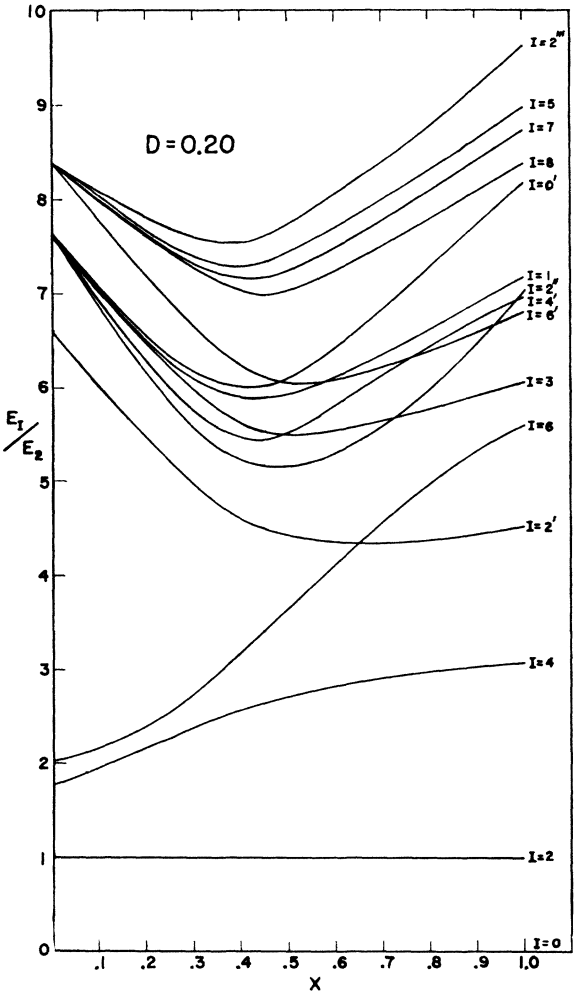

FIG. 1. The computed variation of the ratio of the energy of the level of spin $I(E_I)$ to the energy of the first excited state (E_2). The calculation is for $D=0.20$ and $0\leqslant x\leqslant 1.0$. [The shell-model states are $((\frac{7}{2})^2)_{0,2,4,6}$ with surface interaction included.]

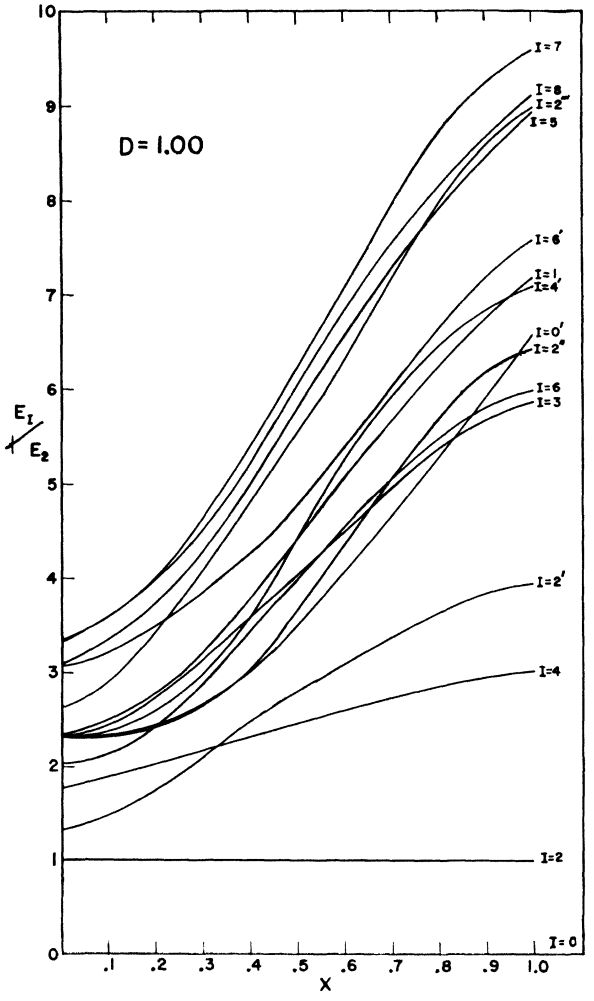

FIG. 2. The same spectra as Fig. 1 with a different choice of D. Here $D=1.00$ and $0\leqslant x\leqslant 1.0$.

[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 27, No. 16 (1953).

vibrational potential energy, g_R is the gyromagnetic ratio of the collective motion usually taken as Z/A, and g_j is the gyromagnetic ratio of the single particle defined as $g_j=\{g_l\pm[1/(2l+1)](g_s-g_l)\}$ for $j=l\pm\frac{1}{2}$. For protons $g_l=1$ and $g_s=5.585$ while for neutrons $g_l=0$ and $g_s=-3.826$. The single-particle magnetic moment μ is just equal to $jg_j\mu_0$. These calculations are equally valid for either $(f_{7/2}{}^2)_J$ or $(g_{7/2}{}^2)_J$ two-particle states since the collective Hamiltonian has no l dependence. [This, of course, assumes that the two-particle unperturbed energies are taken as the same for $(f_{7/2}{}^2)$ and $(g_{7/2}{}^2)$. This equality depends on the particular choice of two-particle Hamiltonian and, in general, the same two-particle Hamiltonian will not give the same energies for these two shell-model configurations. This could be compensated for by redefining D.] The value of the $M1$ transition rate, however, does depend on the value of l. This comes in through the values of g_j which is different for $f_{7/2}$ and $g_{7/2}$. Table III gives the values of g_j and of (g_j-g_R) for the four possible cases of $j=\frac{7}{2}$. The value of g_R is taken as 0.4 in this table.

Figures 1 through 4 show the energy spectrum for D equal to 0.2, 1.0, 2.0, and 3.0 with x varying between 0 and 1.0. Figures 5 and 6 show the same levels displayed in a different manner, with x set equal to 0.25 and 0.45 and D varying between 0 and 3.0. In all these curves only the following 15 levels were plotted: $0, 0', 1, 2, 2', 2'', 2''', 3, 4, 4', 5, 6, 6', 7, 8$. One or two of the omitted levels occur at lower energies than the plotted results for certain of the choices of D and X. A more complete spectrum is available from the author. The curves demonstrate rapidly varying picture of the spectrum as the parameters are varied. This clearly is caused by the interaction of the vibrational states and the shell-model states. For $D=1.32$ the $((\frac{7}{2})^2)_2$ level is degenerate with the one-phonon $I=2$ level while for $D=2.64$ the $((\frac{7}{2})^2)_2$ level is degenerate with the two-phonon $I=2$ level. This degeneracy accounts for the rather pronounced differences in the spectra computed at $D=1.0$, 2.0, and 3.0. The spectra for $D=3.0$ suffer from the fact that the four-phonon states with spin 5, 6, and 8 were not included in the spectrum (no four phonon states were included for any spin). This distorts the spectrum

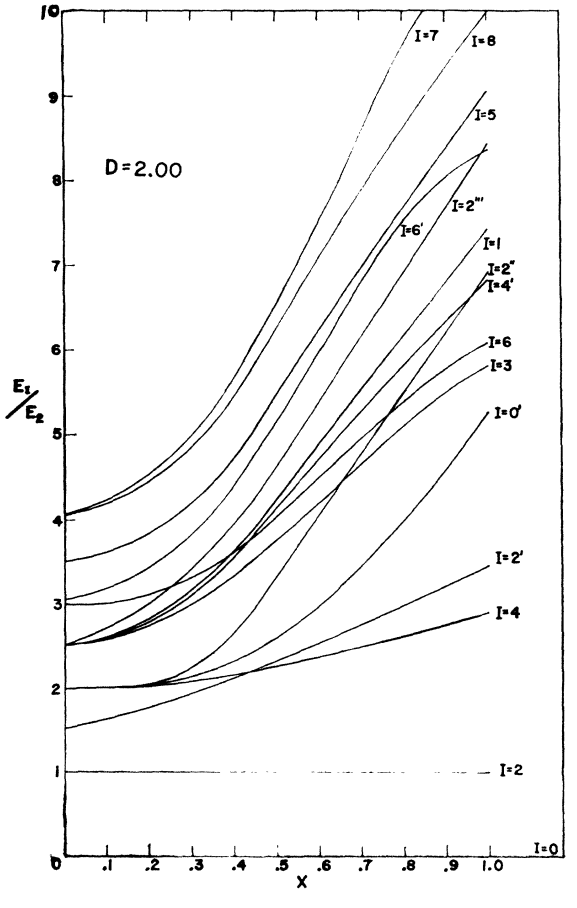

FIG. 3. Here $D=2.00$ and $0\leqslant x\leqslant 1.0$.

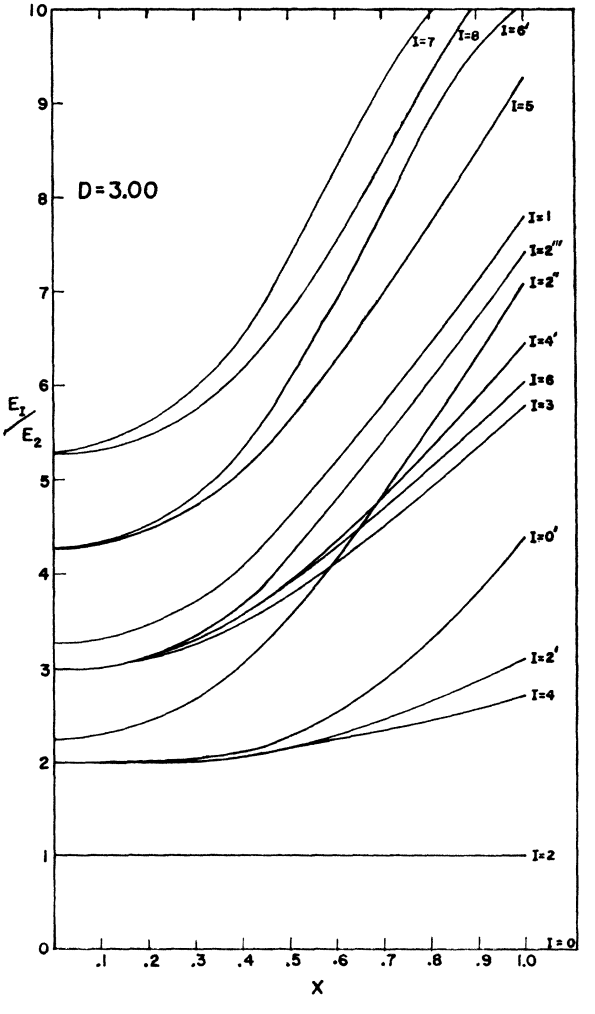

FIG. 4. Here $D=3.00$ and $0\leqslant x\leqslant 1.0$.

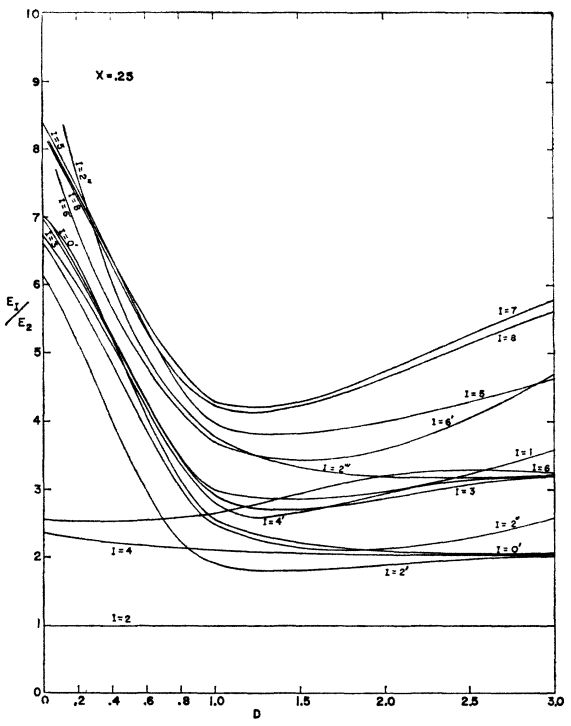

FIG. 5. This represents the variation of the spectra for a constant value of x equal to 0.25. The variable D goes from 0 to 3.0.

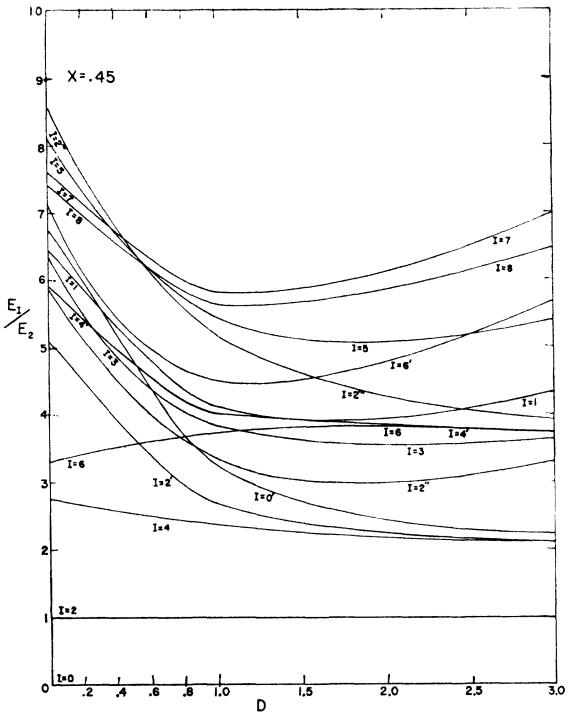

FIG. 6. Here $x=0.45$ and $0 \leqslant D \leqslant 3.0$.

at $x=0$ since these levels would appear at $E_I/E_2=4.0$, while the resulting spectra have levels of these spins at higher energy. The validity of the cutoff at three phonons has been investigated in I, where results are presented for $I=0$ for cutoff at three and four phonons. These results indicate that only small differences occur for values of x less than 1.0.

TABLE IV. Experimental level in even-even nuclei[a] in the $f_{7/2}$ region of the Periodic Table and theoretical results that best fit the observations.

	Ca^{42}		Theory $D=0.2$		Ti^{50}			Theory $D=0.2$		Fe^{54}			Theory $D=0.2$
E	I	E_I/E_2	$x=0.02$		E	I	E_I/E_2	$x=0$		E	I	E_I/E_2	$x=0.02$
0	0^+	0	0		0	0^+	0	0		0	0^+	0	0
1.524	2^+	1	1		1.56	2^+	1	1		1.41	(2^+)	1	1
1.836	0^+	1.21			...					...			
2.424	$2^+(1^+)$	1.59			...					...			
2.752	4^+	1.80	1.80		2.76	4^+	1.77	1.78		2.55	(3,4)	1.81	1.80
3.191	$6^+(5^+)$	2.09	2.05		3.0	6^+	1.92	2.02		2.97	(5,6)	2.10	2.05

	Ca^{44}		Theory $D=0.2$ $x=0.33$		Cr^{52}			Theory $D=0.20$
E	I	E_I/E_2	I	E_I/E_2	E	I	E_I/E_2	$x=0.08$
0	0^+	0	0^+	0	0	0^+	0	0
1.159	2^+	1	2^+	1	1.434	2^+	1	1
1.88	0^+	1.62			2.369	4^+	1.65	
					2.648	(0^+)	1.85	
2.28	(4^+)	1.97			2.77	4^+	1.92	1.92
2.66	2^+	2.30			3.112	6^+	2.17	2.13
(2.86)	(4^+)	2.47	4^+	2.47				
3.04		2.62			3.614	5^+	2.52	
3.03	6^+	2.85	6^+	2.88	3.832	6^+	2.67	
3.35	(2^+)	2.89						
3.58		3.09						
3.66	$(5^+,6^+)$	3.16						

[a] Data were taken from P. C. Rogers and G. E. Gordon (to be published), from *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D. C.), and from L. T. Dillman, J. J. Kraushaar, and J. D. McCullen, Nucl. Phys. (to be published). I am indebted to G. E. Gordon, J. J. Kraushaar, and J. D. McCullen for making these results available to me before publication and for stimulating discussions of the experimental regularities.

IV. CONCLUSIONS

One aim of this study was to match the spectra of $_{20}Ca^{42}$ and $_{22}Ti^{50}$; to some extent this has been successful. The known levels in these nuclei and the corresponding predictions from this work are listed in Table IV.

Note added in proof. The low excited states of Ti^{48} have been experimentally observed by R. A. Ristinen, A. A. Bartlett, and J. J. Kraushaar (to be published). They find the levels with the following energy, spin, and energy ratio (E_I/E_2): (a) 0 keV, 0^+, 0; (b) 983.3 keV, 2^+, 1; (c) 2295.0 keV, 4^+, 2.33; (d) 2430 keV, $(1,2^+)$, 2.48; (e) 3223.9 keV, 4^+, 3.28; (f) 3239.9 keV, (5^+), 3.30; (g) 3340 keV, 6^+, 3.40; (h) 3620, ?, 3.78. This can be compared with the theoretical predictions for $D=1.0$, $X=0.4$ where the following spins and energy ratios (E_I/E_2) are computed: (a) 0^+, 0; (b) 2^+, 1; (c) 4^+, 2.30; (d) 2^+, 2.48; (e) 0^+, 3.00; (f) 2^+, 3.05; (g) 6^+, 3.40; (h) 4^+, 3.58; (i) 1^+, 3.76. I wish to thank the authors for permission to use their results before publication.

(It is also of interest that these results fit many of the levels in Ca^{44}, Cr^{52}, and Fe^{54}. These are also listed in Table IV.)

As can be seen, the second excited state of spin 0^+ in Ca^{42} cannot be matched with any choice of the parameters. An obvious solution to this problem is suggested by the work of Thankappan and Pandya[6] who have used this same model with $1d_{3/2}$ and $2s_{1/2}$ orbitals to fit the levels in Si^{30}. Their results have two shell-model 0^+ states $(d_{3/2}^2)_0$ and $(s_{1/2}^2)_0$. This gives a low-lying 0^+ state for a variety of parameters. Similar calculations are now underway adding $2p_{3/2}$ orbitals to the present calculation. Hopefully these results will give a much better fit to all the levels and perhaps also account for the very large $E0$ transition in[7] Ca^{42} from the 1.84-MeV 0^+ level to the ground state.

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[6] V. K. Thankappan and S. P. Pandya (to be published). I am indebted to S. P. Pandya for making this work available to me before publication.

[7] N. Benczer-Koller, M. Nessin, and T. H. Kruse, Phys. Rev. **123**, 262 (1961).

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Beta-Decay Matrix Elements in Sb^{122}†*‡

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An electronic computer has been used to investigate the six nuclear matrix elements which enter into the 2^- to 2^+ 1.40-MeV beta transition in the decay of Sb^{122}. Data from beta-gamma angular correlation, beta-circularly polarized gamma angular correlation, nuclear orientation, and nuclear resonance experiments were used in this analysis. As a further aid, the Feenberg-Ahrens relations between certain of the nuclear matrix elements were employed to catalog the solutions and to simplify the search problem. In order to discover how the remaining ambiguity of these solutions could most easily be reduced, for each of the solutions calculations were made of the predicted results of all possible experiments on this beta transition. These calculations show how sufficient experimental data can be obtained to determine unambiguously all six nuclear matrix elements. In an appendix all the theoretical formulas which give the experimental observables for a first forbidden 2^- to 2^+ beta transition in terms of the nuclear matrix elements are summarized.

INTRODUCTION

THERE are, in general, six nuclear matrix elements which can contribute to a 2^- to 2^+ first forbidden beta transition.[1] The possibility of experimentally determining so many overlap integrals for the same two nuclear configurations makes these transitions particularly attractive for studying nuclear structure. Although a large amount of both experimental and theoretical work has been reported on transitions of this type, no one has succeeded in finding a unique solution for the six nuclear matrix elements.[2] One of the principal reasons

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* This work was done in part at the Computation Center at the Massachusetts Institute of Technology, Cambridge, Massachusetts.

‡ A preliminary account of part of this work has been given: Bull. Am. Phys. Soc. **7**, 34 (1962).

§ Alfred P. Sloan Research Fellow, 1961–63.

[1] For a recent comprehensive review of first forbidden beta decay see H. A. Weidenmüller, Rev. Mod. Phys. **33**, 574 (1961).

[2] Some recent papers not in reference 1 in which attempts have been made to find the nuclear matrix elements for first forbidden transitions are: Sb^{124}—P. Alexander and R. M. Steffen, Phys. Rev. **124**, 150 (1961); R. M. Steffen, *ibid.* **124**, 145 (1961). Eu^{152}—H. Dulaney, C. H. Braden, and L. D. Wyly, *ibid.* **125**, 1620 (1961). Eu^{154}—S. K. Bhattacherjee and S. K. Mitra, *ibid.* **126**, 1154 (1962). I^{126}, Rb^{84}, As^{74}—D. S. Harmer and M. L. Perlman, *ibid.* **122**, 218 (1961).